No. LVIII.

On the best angles for the sails of a windmill. By John Garnett of New Brunswick, N. Jersey.

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The *angle of weather*, or that angle which the section of the vane, at a given distance from the centre of motion, makes with the plane of its motion, will depend on its proportionate velocity to that of the wind; some mean velocity of which is generally assumed, to which the interior mechanism of the mill is adapted; supposing this at $12\frac{1}{2}$ feet per second, which would be called a *fresh gale*, the Dutch mills, with the sails of 30 feet radius, make about 13 revolutions per minute; in this case, the extremity of the vane moves with nearly three times the velocity of the wind, and consequently, at 10 feet distance with the same velocity as the wind; at 20 feet with twice the velocity, being in a direct proportion to the distance from the centre.

As different *angles of weather* have been given by several writers, as Parent, De Moivre, Maclaurin and Simpson, Smeaton &c. and lately by Mr. Hall Gower, which have been copied in the modern treatises on mechanics, as Gregory, Grey &c. where errors may be of considerable consequence, I will endeavour to shew the true principles, and give a very simple construction which will give the *angle of weather* on either hypothesis.

Let the line WV represent the wind's direction and velocity; SV, the direction and velocity of any section of the vane whose *angle of weather* is required; then WS will be the *relative direction and velocity* of the wind to that part of the sail; and the angle WSV, will be the *limit of the angle of weather*; for, at this angle, the wind's relative direction being parallel to WS, can have no effect, and at any greater angle the vane would be a back sail; the *angle of weather* therefore must be less than WSV; suppose it CSV; then WSC will be the *relative angle of incidence* of the wind on the plane SC; from W draw WC perpendicular to the plane SC, and from C draw CM perpendicular

to WV; then WC will be the relative velocity and proportionate force with which the wind strikes the vane perpendicular to its plane, which force being resolved into two forces WC and cC, the first perpendicular to the plane of motion and therefore of no effect, the last parallel to it and therefore represents the effective force of every particle of wind to turn the vane, when the force perpendicular to the plane is CW; now let any other *angles of weather* as VSB, VSA, be taken, then will WSB, WSA be the *relative angles of incidence*; BW, AW the relative velocity and proportionate force perpendicular to the plane of the sail; and Bb, Aa the effective forces to turn the vanes SB, SA; whence it appears that these forces are ordinates to the chord WV, and if the force of the wind on the vane were in the simple ratio of its relative velocity, or the number of impinging particles were invariable, as is the case in undershot water wheels, as observed by Mr. Waring, in the third volume of the Philosophical Transactions; then the greatest force would be SB, where the *angle of weather* VSB equally divides the *angle of limit* WSV, which agrees with the theorem given by Maclaurin and Simpson, on the above supposition, which to distinguish we may call "*Waring's hypothesis.*" But if the force be as the *square* of the relative velocity; describe a semicircle on SW, and with the radius SD describe the arc D RE cutting the plane SC in R from which draw RF, RG perpendicular to SW, SV. Then the proportionate force on any point C perpendicular to the plane SS will be $\frac{WC^2}{WV^2}$; or (since R F, SN are halves of CW, WV,) $\frac{RF^2}{SN^2}$; and as WC : cC or SR : RG :: (the force perpendicular to SC,) $\frac{RF^2}{SN^2} : \frac{RF^2 \times RG}{SN^2 \times SR}$ the whole *effective force* on each point of the plane SC, which is the same as given by Maclaurin and Simpson, and *agrees with their hypothesis.*

But the same breadth of sail inclined to different angles of weather will not intercept an equal current of wind; the relative current being the parallelogram WZCP to the plane CS,

the particles intercepted will be as CP; or Rp on the plane SR, so that $Rp \times \frac{RF^2 \times RG}{SN^2 \times SR}$ will represent the force on the plane SR, but SN : SD (= SR) : : RF : Rp $= RF \times \frac{SR}{SN}$ therefore the whole effective force on the section SR will be $\frac{RF^3 \times RG}{SN^3}$ which by Simpson's fluxions vol. 2, Prob. 5 page 503, will be a maximum when MN $= \frac{1}{2}$ SN. But if the whole effective force according to *Maclaurin and Simpson* be as $RF^2 \times RG$, (SR and SN being constant) the maximum will then be when MN $\frac{1}{3} =$ SN.

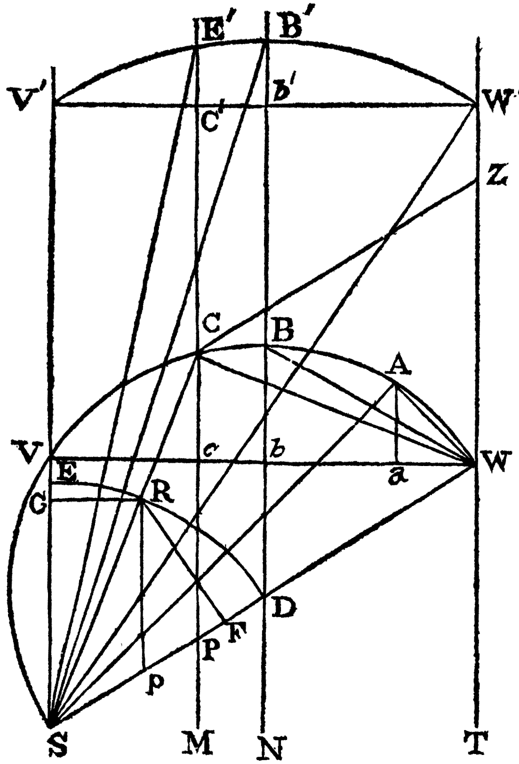
But supposing the angle of weather known for every proportionate velocity of the sail to the wind, it still remains to be determined what that proportion ought to be at the extremity of the sail, as was justly observed by Mr. Smeaton; who, in putting Maclaurin's theory to the test of experiment, *assumed it* as two to one, but he found that by increasing the *angle of weather* three and six degrees, the effect or product was still increasing, although by increasing the *angles of weather* at every part equally they became no longer the angles of Maclaurin; to have made it decisive he should also have taken it as one and an half to one; and as one to one, the forces from the theory continually increasing, as these ratios diminish. Mr. Smeaton also appears to have made an error by estimating the mean velocity of the wind from the distance of the axis of his rotary machine from the centre of his sails; the force being as the squares of the velocities, the mean should have been taken at a greater distance; if this error be corrected, his conclusion that the extremity of the sail should move with 2,7 times the velocity of the wind, will probably be altered to less than double, and from theory a slower motion of the sails appears to be highly advantageous.

The angle of weather on either of these hypotheses is very easily laid down by the following construction, from whence some useful conclusions may be drawn.

Let WV represent the direction and velocity of the wind; SV, SV′ &c. that of the sail at those distances from the centre of motion, and perpendicular to WV; draw the parallel TW′, also NB′ in the middle and MC′ either at $\frac{1}{3}$ the distance between N and S, according to Maclaurin's, or at $\frac{1}{2}$ the distance, by the last hypothesis. Then to find the *angle of weather* corresponding to any velocity or distance TW, draw SW, on which describe a semicircle SVBW, and draw SC, SB, to where it meets the parallels MC′, NB′; then will CSV, be the *angle of weather* according to Maclaurin, if at $\frac{1}{3}$ distance; or according to the last hypothesis, if MC′ be at $\frac{1}{2}$ the distance between N and S; and BSV, equal to *half the angle of limit* WSV, will be the angle of weather according to Waring, if any constant portion of wind could be intercepted; and for Mr. Hall Gower's hypothesis, take TW′, as the whole length of the vane, and *assuming* any angle at the extremity as most advantageous, suppose TW′H = 10 degrees, then any other line drawn from H, to any other distance on TW′, will shew the corresponding *angle of weather*; but this principle will be found evidently erroneous.

The calculation of the different *angles of weather* is also much easier from this construction, than from Maclaurin's theorem for that purpose; for taking WV = 1, as the winds velocity; then the sails' velocity TW is = contangent of the angle of limit = twice the angle of weather according to Waring; and either $\frac{1}{3}$ or $\frac{1}{2}$ the sine of the angle of limit SN, will give the sine of the arc BC, which arc subtracted from the angle of limit (BW =) BV will give the arc VC = twice the *angle of weather* according to either of the two hypotheses.—By this method, the following table is calculated shewing the different angles of weather, and the effective force of the wind to turn the sail; but it must be understood that the proportionate velocity of the extreme part of the sail to the wind can be assumed at pleasure, if the interior mechanism be adapted to it; the greater the angle of weather the less will be the sails' velocity, but the force greater; and also, on either hypothesis, if the same *angle of weather* be assumed at the extremity, the difference of all the other angles would in practice be imperceptible, until we approach towards the centre, where they approximate to either 30°, 35° 16′, 45° or 90°; the last, evidently erroneous.

Velocity of Sail V. Wind=100	M N=½ S N.	Maclaurin M N=1-3 S N	Waring= ½ angle of limit.	Gower	Smeaton.	M N=½ S N.	Maclaurin.	Waring.	Gower.
	ANGLES OF WEATHER.					EFFECTIVE FORCE. R F3 × R G ÷ S N3			
	° ′	° ′	° ′	° ′					
0	30 0	35 16	45 0	90 0		,3248	,3142	,2500	,0000
25	24 20	28 33	38 59	63 26	° ′	,2300	,2238	,1674	,0151
50	18 26	23 4	31 43	45 0	18 0	,1563	,1488	,1067	,0305
75	14 46	18 50	26 34	33 41		,1190	,1128	,0781	,0400
100	12 10	15 41	22 30	26 34	19 0	,0950	,0897	,0607	,0503
125	10 14	13 15	19 20	21 48		,0786	,0758	,0492	,0371
150	8 48	11 31	16 51	18 26	18 0	,0668	,0629	,0413	,0338
175	7 42	9 46	14 53	15 57		,0581	,0556	,0356	,0306
200	6 48	9 0	13 15	14 2	16 0	,0511	,0479	,0311	,0275
250	5 33	7 21	10 54	11 18	12 30	,0414	,0388	,0250	,0229
300	4 40	6 12	9 13	9 28	7	,0346	,0325	,0208	,0197



REMARKS.

1st. The most material consequence to be derived from the above table is the great diminution of the effective force of the wind, as the velocity of the sail increases; which shews, that the sail-cloth should be placed as near the centre as possible, only observing that the *wind must have a free escapement*; for a square foot of sail, moving with half the velocity of the wind, appears to have three times the effective power as when moving with double the wind's velocity; for *the power of the lever when time is considered must be out of the calculation*; this also agrees with Mr. Smeaton's experiments, who found, that by enlarging the breadth of his sails, he gained more than by increasing the radius. Probably the extremity of the sail should not exceed the velocity of the wind; and as this will increase the *angle of weather*, the wind will have a more free escapement, and its reflections be less liable to impede the following sail: the angle of reflection is easily seen from the relative angle of incidence, DSR.

2dly. That Mr. Hall Gower's hypothesis is highly disadvantageous; for by approximating to 90° at the centre it has the least power, where it should have the most.

3dly. It appears evident from theory, and all Mr. Smeaton's experiments, that the greater the angle of weather the slower will be the motion; therefore if by *any simple contrivance* the *angles of weather* could be occasionally altered, it would be the best mode of making the revolutions more uniform, and even of stopping them altogether: I am now making an experiment at large on this method.

4thly. Although the forces appear greatest in the first column, from taking $RF^3 \times RG \div SN^3$ as the measure, yet if the measure had been taken $\frac{RF^2 \times RG}{SN^2 \times SR}$ according to Maclaurin, then the second column had shewn the greatest forces, and the third column, if Bb was the true measure—but on no hypothesis could Gower have any competition.

N. B. $RF^2 \times RG$ is a maximum when $WC \times cC$ is a maximum, and $RF^3 \times RG$ is a maximum, when $cW^{\times}cC$ is a maxium—the first when Wc is $\frac{2}{3}$ of Wv, the last when Wc is $\frac{3}{4}$ of Wv; the greatest right-angled triangle in the segment VBW.